

# Cosmology of a Lorentz violating Galileon theory

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**Abstract.** We modify the scalar Einstein-aether theory by breaking the Lorentz invariance of a gravitational theory coupled to a Galileon type scalar field. This is done by introducing a Lagrange multiplier term into the action, thus ensuring that the gradient of the scalar field is time-like, with unit norm. The theory can also be considered as an extension to the mimetic dark matter theory, by adding some derivative self interactions to the action, which keeps the equation of motion at most second order in time derivatives. The cosmological implications of the model are discussed in detail. In particular, for pressure-less baryonic matter, we show that the universe experiences a late time acceleration. The cosmological implications of a special coupling between the scalar field and the trace of the energy-momentum tensor are also explored.

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## 1 Introduction

Lorentz invariance seems to be a fundamental property of nature which has been tested within the context of the standard model of particle physics with a very high accuracy [1]. However, in the gravity sector, Lorentz invariance cannot be tested to such a high precision. All the gravitational experiments put a mild bound on the validity of Lorentz invariance [2]. In the dark sector of the universe, we have some constraints on the breaking of Lorentz invariance in dark matter [3] and dark energy sectors [4]. For a very good review, see [5]. This means that one can build a Lorentz invariant theory which breaks the Lorentz invariance but satisfy the experimental data. In this sense, it would be plausible to have a gravitational theory, describing the large scale structure and dynamics of the universe which breaks Lorentz invariance.

Another motivation for considering theories that break Lorentz invariance comes from quantum gravity. A very interesting example of such a theory is the Horava-Lifshitz gravity where adding higher order Lorentz violating terms to the action together with different scaling dimensions for space and time makes the theory power counting renormalizable at the ultra-violet level [6]. In the infrared region, in the Horava-Lifshitz theory, the scaling dimension of time becomes unity. Therefore, the Horava-Lifshitz theory can in principle be written in a generally covariant form [7]. However, due to a fixed foliation of space-time into space-like hypersurfaces, the theory should break Lorentz invariance dynamically.

One of the best candidates for a generally covariant gravitational theory with a preferred time-like direction is the Einstein-aether theory, proposed in [8]. The preferred direction can be imposed to the theory by introducing a time-like vector field to the action through a Lagrange multiplier. The action of the theory then becomes

$$S_{ae} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ R + K_{\lambda\sigma}^{\mu\nu} \nabla_\mu u^\lambda \nabla_\nu u^\sigma + \lambda (u_\mu u^\mu + 1) \right] + S_m, \quad (1.1)$$

where  $\lambda$  is a Lagrange multiplier and the tensor  $K_{\lambda\sigma}^{\mu\nu}$  is defined as

$$K_{\lambda\sigma}^{\mu\nu} = c_0 g^{\mu\nu} g_{\lambda\sigma} + c_1 \delta_\sigma^\mu \delta_\lambda^\nu + c_2 \delta_\lambda^\mu \delta_\sigma^\nu + c_3 u^\mu u^\nu g_{\lambda\sigma}, \quad (1.2)$$

where  $c_i$ ,  $i = 0, 1, 2, 3$  are the dimensionless free parameters [8]. The stability of the theory has also been studied and it turns out that the theory has a non-empty region in the parameter space in which no instabilities occur [9]. The PPN parameters were also obtained [10] and it was shown that all PPN parameters are identical to the standard GR if one imposes the conditions  $c_2 = (-2c_1^2 - c_1 c_3 + c_3^2)/3c_1$  and  $c_4 = -c_3^2/c_1$  respectively, for a discussion of this issue see also [11].

The other interesting fact about the Einstein-aether theory is that it can be considered as a covariant version of the Horava-Lifshitz theory in the infrared sector. In fact, one can prove that the above theory is identical to the non-projectable Horava-Lifshitz theory for a special case of the aether field,  $u_\mu = N \nabla_\mu T$ , where  $N$  is a normalization factor [12]. However, the action (1.1) can only produce the lowest terms of the Horava-Lifshitz theory and the search for a generally covariant theory, which produces higher order terms of the Horava-Lifshitz theory is still a subject of research.

It will also be interesting if one could write an action for a scalar field with a preferred time-like direction. In this case the covariant derivative of the scalar field can play the role of a time-like vector. In [13] the authors have suggested such a gravitational model by substituting  $u_\mu$  by  $\nabla_\mu \phi$  in the action (1.1). However, this choice separates only the longitudinal mode of the vector field and also implies that  $\nabla_\mu \phi$  has a unit norm. It was shown in [14] that the theory proposed in [13] is identical to the projectable Horava-Lifshitz gravity which has instabilities [15, 16]. The problem lies in the aether action itself, where substituting the aether field with  $\nabla_\mu \phi$  produces some fourth order derivative terms. These higher order derivative terms will produce ghost instabilities to the action. Hence, in order to write a scalar action which breaks the Lorentz invariance dynamically, one should introduce a kinetic term for the scalar field which has no instabilities.

One of the main motivations of the present paper is to write down an action for a “healthy” scalar-aether theory. We first note that in writing the action (1.1), the authors have introduced the most general canonical kinetic term for a vector field. In the scalar case, the canonical kinetic term is of the form  $\partial_\mu \phi \partial^\mu \phi$ , which is the same as the term used to break the Lorentz invariance. In fact, one adds a term  $\lambda(\partial_\mu \phi \partial^\mu \phi + 1)$  to the action where  $\lambda$  is a Lagrange multiplier. In this paper, we want to add some higher order kinetic interactions to the theory in a way that the resulting theory has no Ostrogradski instability. In this sense one should add some healthy higher order derivative interactions to the action. Such terms are known as Galileons [17].

The Galileons are scalar fields where having even higher than second order time derivatives in the action produce second order field equations. The Galileons were originally written in flat Minkowski space [17] where they have an additional symmetry

$$\phi \rightarrow \phi + b_\mu x^\mu + c, \quad (1.3)$$

which is lost in the covariant version [18]<sup>1</sup>. In order to have a covariant a Galileon theory, one can replace the partial derivatives with the covariant derivatives. However, because of the

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<sup>1</sup>The most general theory of two graviton degrees of freedom (dof) plus a scalar dof writing in terms of curvature invariants and the covariant derivatives of the scalar field was first introduced by Horndeski in [19], and independently rediscovered in [20]. The covariant Galileon theory which we are using in this paper is then a special case of Horndeski theory.

non-commutativity of the covariant derivatives, one obtains some higher order terms which produce Ostrogradski instabilities in the curved background. In order to remove these non-healthy terms one should add to the action some new terms which do not respect the Galilean symmetry (1.3). This will produce a covariant Galileon action [18]. One can also generalize the Galileons by substituting the coefficients of each term with an arbitrary function of the scalar field [22].

Recently, a very interesting model for dark matter sector of the universe was proposed in the literature, known as the “mimetic dark matter” [23]. The model has originally suggested a new action for a conformally invariant gravitational theory. In this sense, the authors have added a scalar field to the theory and built the gravitational action using the effective metric

$$g_{\mu\nu} = (\bar{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi) \bar{g}_{\mu\nu}. \quad (1.4)$$

The theory is obviously invariant under the change  $\bar{g}_{\mu\nu} \rightarrow \Omega^2 \bar{g}_{\mu\nu}$ , with  $\Omega$  an arbitrary function. One can easily observe that in this theory the scalar field  $\phi$  is subject to the constraint  $g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = -1$ , which implies that the vector  $u_\mu = \nabla_\mu \phi$  is time-like [23, 24]. So the vector field  $u_\mu$  will be tangent to time-like geodesics, and will correspond to the 4-velocity of a dust particle. This geometrical dust corresponds to dark matter. The mimetic theory can then explain the dark matter content of the universe as a consequence of the constraint equation  $g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = -1$ . This also happens in the scalar Einstein-aether theory where one adds a similar constraint to the action through a Lagrange multiplier. This suggests that the model introduced in the present paper can be considered as an extension of the mimetic dark matter theory with the addition of the most general higher derivative self interaction which limits the equations of motion to second order. Such a modification makes dark matter sector of the universe imperfect [25]. We should note that one can generalize the constraint equation by rewriting it as  $g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = -\mu(\phi)^2$  [26]. This condition implies that the vector field  $u_\mu = \partial_\mu \phi / \mu$  is tangent to time-like geodesics, corresponding to the 4-velocity of dark matter.

The present paper is organized as follows. In Section 2 we introduce the action of the Lorentz violating Galileon theory and obtain the equations of motion. In Section 3 we study the cosmological implications of the theory, showing that it can explain the accelerated expansion of a matter dominated universe, having only zero pressure dust in the energy-momentum of the universe. In Section 4 we add a special interaction between the aether field and the trace of the energy momentum tensor of ordinary matter and study the cosmology of this modified gravity model. Last Section is devoted a discussion of our results.

## 2 Generalized scalar Einstein-aether theory

We propose a generalization of the scalar Einstein-aether theory based on a gravitational action of the form

$$S = \int d^4x \sqrt{-g} \left[ \kappa^2 R + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4 + \alpha_5 \mathcal{L}_5 + \lambda(\phi_\mu \phi^\mu + 1) + \mathcal{L}_m \right], \quad (2.1)$$

where  $\mathcal{L}_m$  is the matter Lagrangian and  $\phi_\mu \equiv \nabla_\mu \phi$ . In the above action we have dropped the tadpole Galileon term and also absorbed the quadratic term into the term  $\lambda(\phi_\mu \phi^\mu + 1)$ .

We have also introduced the terms  $\mathcal{L}_i$ ,  $i = 1, 2, 3$ , defined as [18]

$$\mathcal{L}_3 = (\phi_\alpha \phi^\alpha) \square \phi, \quad (2.2)$$

$$\mathcal{L}_4 = (\phi_\alpha \phi^\alpha) \left[ 2(\square \phi)^2 - 2\phi_{\mu\nu} \phi^{\mu\nu} - \frac{1}{2}(\phi_\mu \phi^\mu) R \right], \quad (2.3)$$

$$\mathcal{L}_5 = (\phi_\alpha \phi^\alpha) \left[ (\square \phi)^3 - 3(\phi_{\mu\nu} \phi^{\mu\nu}) \square \phi + 2\phi_\mu{}^\nu \phi_\nu{}^\rho \phi_\rho{}^\mu - 6\phi_\mu \phi^{\mu\nu} G_{\nu\rho} \phi^\rho \right]. \quad (2.4)$$

The action (2.1) is similar to the action for mimetic dark matter with derivative interaction terms for a scalar field [24].

Variation of the action with respect to  $\lambda$  gives

$$\phi_\mu \phi^\mu = -1, \quad (2.5)$$

which, as was mentioned before, defines a preferred direction for the space-time, and dynamically breaks the Lorentz symmetry of the theory. After using Eq. (2.5) and its derivative

$$\phi^\mu \phi_{\mu\nu} = 0, \quad (2.6)$$

we obtain the reduced form of the metric field equations as

$$\kappa^2 G_{\mu\nu} = T_{\mu\nu} + \alpha_3 T_{\mu\nu}^3 + \alpha_4 T_{\mu\nu}^4 + \alpha_5 T_{\mu\nu}^5 - \lambda \phi_\mu \phi_\nu, \quad (2.7)$$

where  $T_{\mu\nu}$  is the ordinary matter energy momentum tensor defined as

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}},$$

and we have defined

$$T_{\mu\nu}^3 = -\phi_\mu \phi_\nu \square \phi, \quad (2.8)$$

$$T_{\mu\nu}^4 = -2\phi_{\mu\nu} \square \phi + 2\phi_{\mu\rho} \phi^\rho{}_\nu + ((\square \phi)^2 - \phi_{\rho\sigma} \phi^{\rho\sigma})(g_{\mu\nu} - 2\phi_\mu \phi_\nu) \\ - \phi_\mu \phi_\nu R + 2\phi^\rho (R_{\rho\mu} \phi_\nu + R_{\rho\nu} \phi_\mu) + \frac{1}{2} G_{\mu\nu} - 2\phi_\rho R^{\rho\sigma} \phi_\sigma g_{\mu\nu} + 2\phi^\rho \phi^\sigma R_{\mu\rho\nu\sigma}, \quad (2.9)$$

and

$$T_{\mu\nu}^5 = -3((\square \phi)^2 - \phi_{\sigma\lambda} \phi^{\sigma\lambda}) \phi_{\mu\nu} + \square \phi (6\phi_{\mu\sigma} \phi^\sigma{}_\nu - \frac{3}{2} \phi_\mu \phi_\nu R) + 3\phi^\sigma \square \phi (R_{\sigma\mu} \phi_\nu + R_{\sigma\nu} \phi_\mu) \\ + 3\square \phi \phi^\sigma \phi^\lambda R_{\mu\sigma\nu\lambda} - 6\phi_{\mu\sigma} \phi^{\sigma\rho} \phi_{\rho\nu} + 3\phi_{\sigma\lambda} R^{\sigma\lambda} \phi_\mu \phi_\nu - 3\phi_\sigma R^{\sigma\lambda} (\phi_{\lambda\mu} \phi_\nu + \phi_{\lambda\nu} \phi_\mu) \\ - 3\phi^\sigma \phi^{\lambda\rho} (R_{\mu\lambda\sigma\rho} \phi_\nu + R_{\nu\lambda\sigma\rho} \phi_\mu) + 3\phi^\sigma \phi^\lambda (R_{\mu\sigma\lambda\rho} \phi^\rho{}_\nu + R_{\nu\sigma\lambda\rho} \phi^\rho{}_\mu) + 3\phi_\sigma \phi_\lambda \phi_{\rho\kappa} R^{\sigma\rho\lambda\kappa} g_{\mu\nu} \\ + \left( (\square \phi)^3 - 3\square \phi (\phi_{\rho\sigma} \phi^{\rho\sigma}) + 2\phi_{\rho\sigma} \phi^{\sigma\lambda} \phi_\lambda{}^\rho \right) (g_{\mu\nu} - \phi_\mu \phi_\nu) + 3(\phi_\sigma R^{\sigma\lambda} \phi_\lambda) (\phi_{\mu\nu} - \square \phi g_{\mu\nu}). \quad (2.10)$$

The aether field equation of motion can be written as

$$\alpha_3 \mathcal{E}_3 + \alpha_4 \mathcal{E}_4 + \alpha_5 \mathcal{E}_5 - 2\nabla_\mu (\lambda \phi^\mu) = 0, \quad (2.11)$$

where we have defined

$$\mathcal{E}_3 = -2(\Box\phi)^2 + 2R_{\mu\nu}\phi^\mu\phi^\nu + 2\phi_{\mu\nu}\phi^{\mu\nu}, \quad (2.12)$$

$$\begin{aligned} \mathcal{E}_4 = & -4(\Box\phi)^3 - 8\phi_{\mu\nu}\phi^{\nu\sigma}\phi_\sigma{}^\mu + 12\Box\phi(\phi_{\mu\nu}\phi^{\mu\nu}) - 2(\Box\phi)R \\ & + 8(\Box\phi)\phi_\mu R^{\mu\nu}\phi_\nu + 4\phi_{\mu\nu}R^{\mu\nu} - 8\phi_\mu\phi_\nu\phi_{\sigma\rho}R^{\mu\rho\nu\sigma}, \end{aligned} \quad (2.13)$$

and

$$\begin{aligned} \mathcal{E}_5 = & -2(\Box\phi)^4 + 3(\Box\phi)^2 \left( 4\phi_{\mu\nu}\phi^{\mu\nu} - R + 2\phi_\mu R^{\mu\nu}\phi_\nu \right) - 16\Box\phi(\phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho{}^\mu) + 12(\Box\phi)\phi_{\mu\nu}R^{\mu\nu} \\ & - 12(\Box\phi)\phi_\mu\phi_\nu\phi_{\rho\sigma}R^{\mu\rho\nu\sigma} - 6(\phi_{\mu\nu}\phi^{\mu\nu})(\phi_{\rho\sigma}\phi^{\rho\sigma}) + 12\phi_{\mu\nu}\phi^{\nu\rho}\phi_{\rho\sigma}\phi^{\sigma\mu} + 3(\phi_{\mu\nu}\phi^{\mu\nu})R \\ & - 6(\phi_{\mu\nu}\phi^{\mu\nu})(\phi_\rho R^{\rho\sigma}\phi_\sigma) - 12\phi_{\nu\rho}R^{\rho\sigma}\phi_\sigma{}^\nu - 6\phi_{\nu\rho}\phi_{\sigma\lambda}R^{\nu\sigma\rho\lambda} + 12\phi_\mu\phi_\nu\phi_{\rho\sigma}\phi^\sigma{}_\lambda R^{\mu\rho\nu\lambda} \\ & + 3(\phi_\nu R^{\nu\rho}\phi_\rho)R - 6\phi_\nu R^{\nu\rho}R_{\rho\sigma}\phi^\sigma - 6\phi_\nu\phi_\rho R_{\sigma\lambda}R^{\nu\sigma\rho\lambda} + 3\phi_\nu\phi_\rho R^\nu{}_{\sigma\kappa\lambda}R^{\rho\sigma\kappa\lambda}. \end{aligned} \quad (2.14)$$

Note that the energy-momentum tensors  $T_{\mu\nu}^i$  have the property that their covariant derivatives become proportional to  $\mathcal{E}_i$  [17]

$$\nabla^\mu T_{\mu\nu}^i = \frac{1}{2}\phi_\nu \mathcal{E}_i, \quad i = 1, 2, 3. \quad (2.15)$$

So, the covariant derivative of the equation (2.7) is reduced to

$$\nabla^\mu T_{\mu\nu} = -\frac{1}{2}\phi_\nu [\alpha_3 \mathcal{E}_3 + \alpha_4 \mathcal{E}_4 + \alpha_5 \mathcal{E}_5 - 2\nabla_\mu(\lambda\phi^\mu)]. \quad (2.16)$$

The energy-momentum tensor of ordinary matter is then conserved if the equation of motion for the aether field is satisfied. In the following we will assume that the energy momentum tensor of ordinary matter is conserved  $\nabla^\mu T_{\mu\nu} = 0$ .

### 3 Cosmological implications

In this Section we study the cosmological implications of the Lorentz violating Galileon theory. We will restrict our study to homogeneous and isotropic cosmological models with the line element given by the flat Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2. \quad (3.1)$$

We also assume that the aether field and the Lagrange multiplier are homogeneous and therefore have the form  $\phi = \phi(t)$  and  $\lambda = \lambda(t)$ , respectively. We also assume that the matter energy-momentum tensor, describing the matter content of the universe, has a perfect fluid form

$$T^\mu{}_\nu = \text{diag}[-\rho(t), p(t), p(t), p(t)], \quad (3.2)$$

where  $\rho(t)$  is the matter energy density, while  $p(t)$  represents the thermodynamic pressure.

### 3.1 The cosmological evolution equations

Due to the choice of the geometry as homogeneous and isotropic, the constraint equation Eq. (2.5) completely determines the aether field  $\phi$  as

$$\phi = t + c_1, \quad (3.3)$$

where  $c_1$  is an arbitrary integration constant. For a homogeneous and isotropic cosmological model, Eq. (3.3) is obvious since the FRW metric already has a time-like preferred direction  $\partial/\partial t$ , and the aether vector should be identical to that direction up to a shift.

The metric and scalar field equations can then be obtained from Eqs. (2.7) and (2.11) as

$$\frac{3}{2}(15\alpha_4 + 2\kappa^2)H^2 - 21\alpha_5H^3 - 3\alpha_3H - \rho + \lambda = 0, \quad (3.4)$$

$$(3\alpha_4 + 2\kappa^2 - 6\alpha_5H)\dot{H} + \frac{3}{2}(3\alpha_4 + 2\kappa^2)H^2 - 6\alpha_5H^3 + p = 0, \quad (3.5)$$

and

$$6(15\alpha_5H^2 - 12\alpha_4H + \alpha_3)\dot{H} + 90\alpha_5H^4 - 108\alpha_4H^3 + 18\alpha_3H^2 - 2(3\lambda H + \dot{\lambda}) = 0. \quad (3.6)$$

In Eqs. (3.4) - (3.6)  $H = H(t) = \dot{a}(t)/a(t)$  denotes the Hubble parameter.

One can solve the aether equation for the Lagrange multiplier to obtain

$$\lambda = 3H(\alpha_3 - 6\alpha_4H + 5\alpha_5H^2) + \frac{c_2}{a^3}, \quad (3.7)$$

where  $c_2$  is an arbitrary constant of integration. Substituting Eq. (3.7) into Eqs. (3.4) and (3.5), one obtains

$$\frac{3}{2}(2\kappa^2 + 3\alpha_4)H^2 - 6\alpha_5H^3 + \frac{c_2}{a^3} - \rho = 0, \quad (3.8)$$

$$(3\alpha_4 + 2\kappa^2 - 6\alpha_5H)\dot{H} + \frac{3}{2}(3\alpha_4 + 2\kappa^2)H^2 - 6\alpha_5H^3 + p = 0. \quad (3.9)$$

One can easily see that  $\alpha_5$  introduces higher order Hubble parameter to the Friedmann equation and  $\alpha_4$  modifies the gravitational constant of the theory. Solving Eq. (3.8) for  $\rho$  gives

$$\rho = \frac{3}{2}H^2(3\alpha_4 + 2\kappa^2 - 4\alpha_5H) + \frac{c_2}{a^3}. \quad (3.10)$$

One of the most common equations of state, which has been used extensively to study the properties of compact objects at high densities is the linear barotropic equation of state, with  $p = \omega\rho$  with  $\omega = \text{constant} \in [0, 1]$ . Assuming the equation of state of the ordinary matter as having a linear barotropic form and substituting Eq. (3.10) to Eq. (3.9), we obtain the basic cosmological evolution equation of our model as

$$(2\kappa^2 + 3\alpha_4 - 6\alpha_5H)\dot{H} + \frac{3}{2}(\omega + 1)(2\kappa^2 + 3\alpha_4 - 4\alpha_5H)H^2 + \frac{\omega c_2}{a^3} = 0. \quad (3.11)$$

### 3.2 The general dynamics of the SEA cosmological models

In order to obtain a simpler form of Eq. (3.11) we rescale the parameters  $\alpha_4$ ,  $\alpha_5$  and  $c_2$  so that

$$\alpha_4 = \frac{2\kappa^2}{3}m, \quad \alpha_5 = \frac{\kappa^2}{3}n_1, \quad c_2 = 2\kappa^2s_1, \quad (3.12)$$

where  $m$ ,  $n_1$ ,  $s_1$  are constants. Then Eq. (3.11) takes the form

$$(1 + m - n_1H)\dot{H} + \frac{3}{2}(\omega + 1) \left[ 1 + m - \frac{2}{3}n_1H \right] H^2 + \frac{\omega s_1}{a^3} = 0. \quad (3.13)$$

We introduce as an independent variable the redshift  $z$ , defined as  $1 + z = 1/a$ . Therefore

$$\frac{dH}{dt} = \frac{dH}{dz} \frac{dz}{dt} = -(1 + z)H \frac{dH}{dz}. \quad (3.14)$$

Moreover, we represent  $H(z)$  as  $H(z) = H_0h(z)$ , where  $H_0$  is the present value of the Hubble parameter. By rescaling again the coefficients  $n_1$  and  $s_1$  so that

$$n_1 = \frac{n}{H_0}, \quad s = \frac{s_1}{H_0^2}, \quad (3.15)$$

we obtain the basic evolution equation of the Hubble parameter as

$$(1 + z)h(z) [1 + m - nh(z)] \frac{dh}{dz} = \frac{3}{2}(\omega + 1) \left[ 1 + m - \frac{2}{3}nh(z) \right] h^2(z) + \omega s(1 + z)^3. \quad (3.16)$$

By rescaling the density as  $\rho(z) = 2\kappa^2H_0^2r(z)$  one obtains

$$r(z) = \frac{3}{2} \left[ 1 + m - \frac{2}{3}nh(z) \right] h^2(z) + s(1 + z)^3. \quad (3.17)$$

By rescaling the constant  $\alpha_3$  as  $\alpha_3 = 2\kappa^2uH_0$ , where  $u$  is a constant, and the Lagrange multiplier  $\lambda$  as  $\lambda = 2\kappa^2H_0^2\Lambda$ , we obtain

$$\Lambda(z) = 3h(z) \left[ u - 2mh(z) + \frac{5}{6}nh^2(z) \right] + s(1 + z)^3. \quad (3.18)$$

As an indicator of the accelerated expansion we introduce the deceleration parameter, defined as

$$q = \frac{d}{dt} \frac{1}{H(t)} - 1 = -\frac{\dot{H}(t)}{H^2(t)} - 1 = (1 + z) \frac{1}{H(z)} \frac{dH(z)}{dz} - 1. \quad (3.19)$$

With the use of Eq. (3.13) it follows that the deceleration parameter can be expressed as

$$q = \frac{3}{2}(\omega + 1) \frac{2\kappa^2 + 3\alpha_4 - 4\alpha_5H}{2\kappa^2 + 3\alpha_4 - 6\alpha_5H} + \frac{\omega c_2}{a^3H^2(2\kappa^2 + 3\alpha_4 - 6\alpha_5H)} - 1, \quad (3.20)$$

or, equivalently



$$q = \frac{3}{2}(\omega + 1) \frac{1 + m - (2n_1/3)H}{1 + m - n_1H} + \frac{\omega s_1}{a^3 H^2 (1 + m - n_1H)} - 1. \quad (3.21)$$

As a function of redshift the deceleration parameter is obtained as

$$q(z) = \frac{3}{2}(\omega + 1) \frac{1 + m - (2/3)nh(z)}{1 + m - nh(z)} + \frac{\omega s(1+z)^3}{h^2(z)[1 + m - nh(z)]} - 1. \quad (3.22)$$

The sign of the deceleration parameter indicates the nature of the expansionary evolution. If  $q > 0$ , the cosmological expansion is decelerating, while negative values of  $q$  indicate an accelerating dynamics.

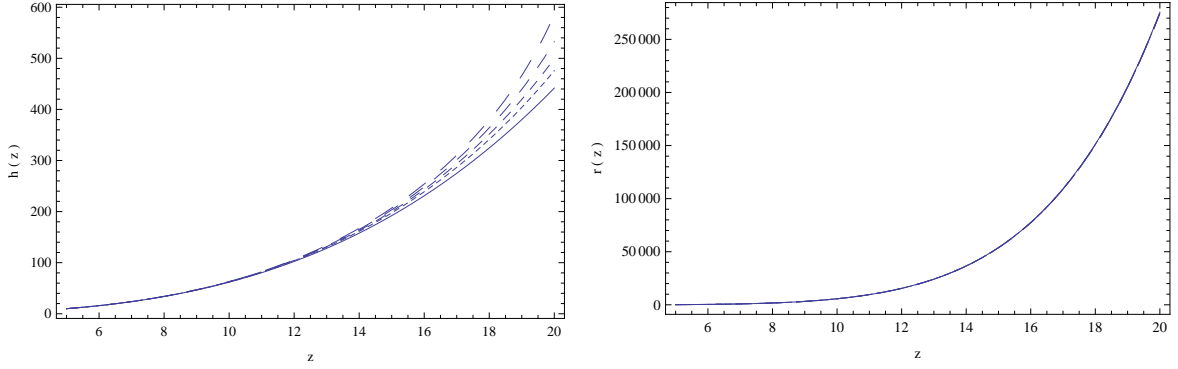
In the following we consider the cosmological implications of the present model for several equations of state of the cosmological matter. We first investigate the high density phase of the evolution of the universe with matter obeying the stiff and radiation equations of state. We then analyze in detail the behavior of dust (zero thermodynamic pressure) cosmological models.

### 3.3 The stiff and radiation fluid cases

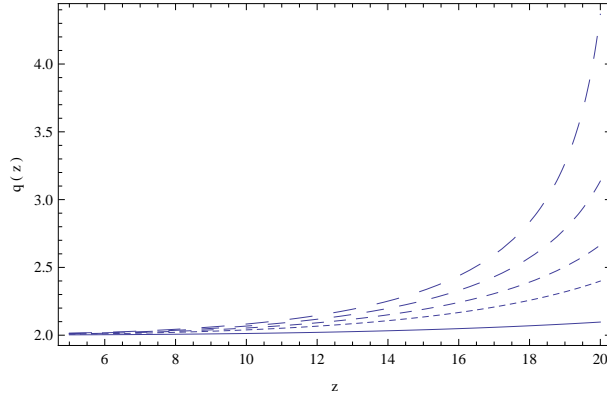
We begin our study of the cosmology of the Lorentz violating Galileon model by analyzing the high density universe described by the Zeldovich (stiff) equation of state and by the radiation equation of state, respectively. The Zeldovich equation of state  $p = \rho$  is valid for densities significantly higher than nuclear densities  $\rho_n$ ,  $\rho > 10\rho_n$ . It can be obtained by constructing a relativistic Lagrangian which allows bare nucleons to interact attractively through scalar meson exchange, and repulsively through the exchange of a slightly more massive vector meson [21]. In the non-relativistic limit both the quantum and classical theories lead to Yukawa-type self-interaction potentials. But at the highest possible matter densities the vector meson exchange dominates. By using a mean field approximation for the nuclear interactions, it follows that in the extreme limit of very high densities the thermodynamic pressure tends to the energy density,  $p \rightarrow \rho$ . In this high density limit the speed of sound  $c_s$  tends to one,  $c_s^2 = dp/d\rho \rightarrow 1$ . Therefore the Zeldovich (stiff fluid) equation of state satisfies the causality condition with the speed of sound less than or equal to the speed of light. The radiation fluid satisfies an equation of state of the form  $p = (1/3)\rho$ .

The variations of the Hubble parameter, energy density and deceleration parameter are represented, as functions of the redshift  $z$ , and for different values of the parameters  $m$ ,  $n$  and  $s$  for a stiff fluid filled universe, in Figs. 1 and 2, while the variation of the same quantities for a radiation filled universe are represented in Figs. 3 and 4, respectively.

In both the stiff and radiation fluid cases we have studied the evolution of the high density universe in the redshift range  $5 \leq z \leq 20$ , with the initial condition  $h(5) = 10$ . The behavior of the cosmological models for both equation of states is similar. For the range of considered parameters  $m$ ,  $n$  and  $s$ , the Hubble function is a monotonically increasing function of  $z$  (monotonically decreasing in time), as is the energy density. The evolution of the deceleration parameter shows a decelerating expansion, with values of the order of  $q(5) \approx 2$  in the case of the stiff fluid universe and with  $q(5) \approx 1$  in the case of the radiation fluid. Of course, modifying the numerical values of  $m$ ,  $n$  and  $s$  may lead to significantly different values of the cosmological parameters.



**Figure 1.** Variation of the Hubble parameter  $h(z)$  (left figure) and of the energy density  $r(z)$  (right figure) as a function of  $z$  for a stiff fluid filled universe, for different values of the parameters  $m, n, s$ :  $m = 0.0001, n = 0.0002, s = 0.0003$  (solid curve),  $m = 0.0004, n = 0.0006, s = 0.0008$  (dotted curve),  $m = 0.0006, n = 0.0008, s = 0.001$  (short dashed curve),  $m = 0.0008, n = 0.001, s = 0.0012$  (dashed curve), and  $m = 0.001, n = 0.0012, s = 0.0014$ , respectively.



**Figure 2.** Variation with respect to  $z$  of the deceleration parameter  $q(z)$  for a stiff fluid filled universe, for different values of the parameters  $m, n, s$ :  $m = 0.0001, n = 0.0002, s = 0.0003$  (solid curve),  $m = 0.0004, n = 0.0006, s = 0.0008$  (dotted curve),  $m = 0.0006, n = 0.0008, s = 0.001$  (short dashed curve),  $m = 0.0008, n = 0.001, s = 0.0012$  (dashed curve), and  $m = 0.001, n = 0.0012, s = 0.0014$ , respectively.

### 3.4 The universe filled with dust

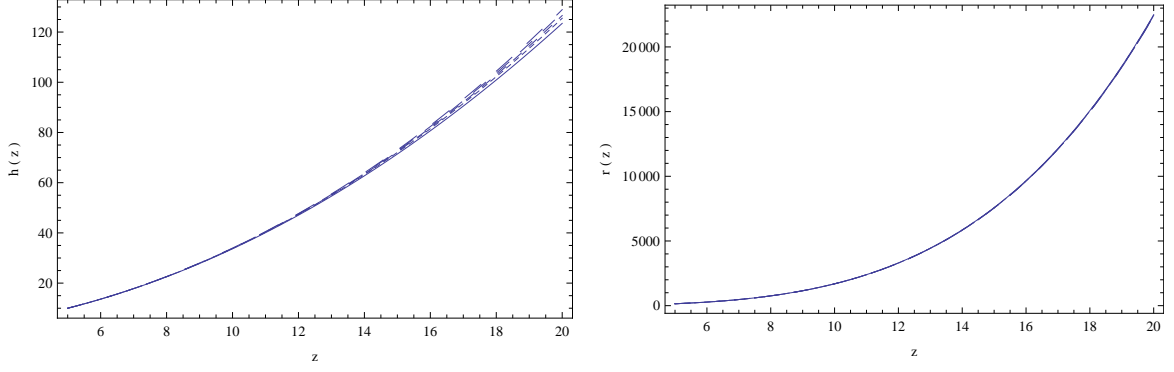
For a universe filled with dust,  $\omega = 0$ , the time evolution equation Eq. (3.13) for the Hubble parameter takes the form

$$(1 + m - n_1 H) \dot{H} + \frac{3}{2} \left[ 1 + m - \frac{2}{3} n_1 H \right] H^2 = 0. \quad (3.23)$$

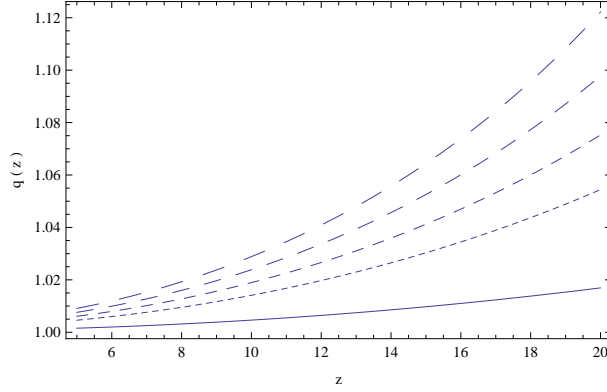
Eq. (3.23) admits a de Sitter type solution  $H = H_0 = \text{constant}$ , corresponding to

$$H_0 = \frac{3(1 + m)}{2n_1}. \quad (3.24)$$

For this choice of parameters the expansion is exponential with the scale factor given by  $a = a_0 e^{H_0 t}$ , with the deceleration parameter having the value  $q = -1$ . During a de Sitter



**Figure 3.** Variation as a function of  $z$  of the Hubble parameter  $h(z)$  (left figure) and of the matter energy density  $r(z)$  (right figure) for a radiation fluid filled universe, for different values of the parameters  $m, n, s$ :  $m = 0.0001, n = 0.0002, s = 0.0003$  (solid curve),  $m = 0.0004, n = 0.0006, s = 0.0008$  (dotted curve),  $m = 0.0006, n = 0.0008, s = 0.01$  (short dashed curve),  $m = 0.0008, n = 0.01, s = 0.012$  (dashed curve), and  $m = 0.001, n = 0.012, s = 0.014$ , respectively.



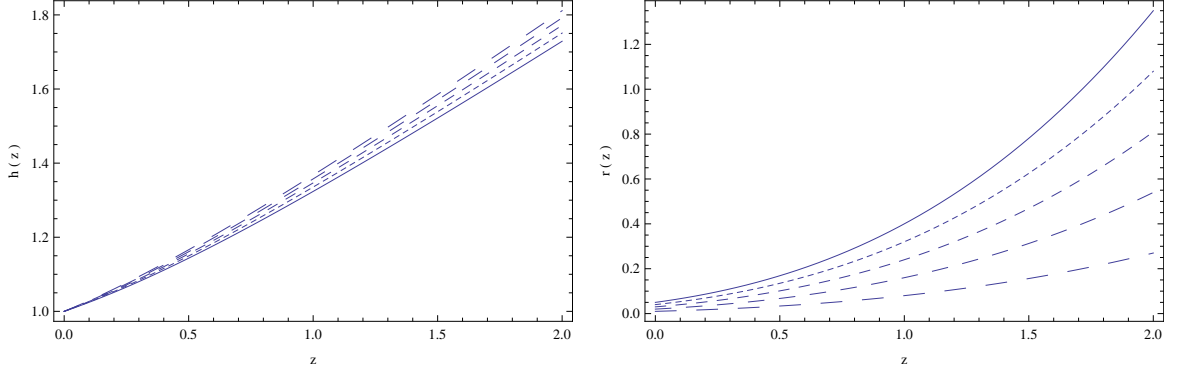
**Figure 4.** Variation with respect to  $z$  of the deceleration parameter  $q(z)$  for a radiation fluid filled universe, for different values of the parameters  $m, n, s$ :  $m = 0.0001, n = 0.0002, s = 0.0003$  (solid curve),  $m = 0.0004, n = 0.0006, s = 0.0008$  (dotted curve),  $m = 0.0006, n = 0.0008, s = 0.01$  (short dashed curve),  $m = 0.0008, n = 0.01, s = 0.012$  (dashed curve), and  $m = 0.001, n = 0.012, s = 0.014$ , respectively.

type phase the energy density of the universe is given by  $\rho = c_2/a^3$ , and it tends exponentially to zero. For arbitrary values of the parameters the general solution of Eq. (3.23) is given by

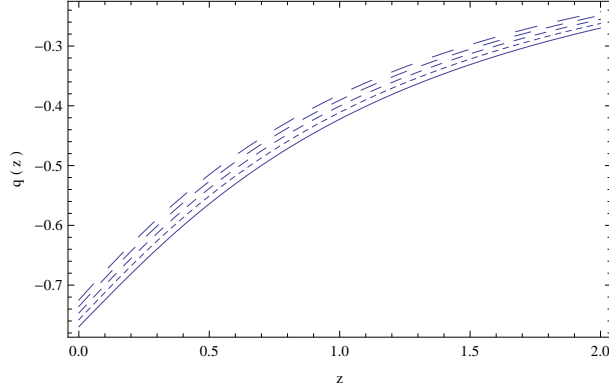
$$\frac{3}{2}(t - t_0) = \frac{1}{H} + \frac{n_1}{3(1+m)} \ln \frac{H}{3 + 2m - 2n_1 H}. \quad (3.25)$$

The variations with respect to the redshift  $z$  of the Hubble function, matter energy density and deceleration parameter for the dust universe are represented in Figs. 5 and 6.

Both the Hubble function and the matter energy density, shown in Figs. 5, are monotonically decreasing functions of time. The deceleration parameter, represented in Fig. 6 has negative values in the range  $0 \leq z \leq 2$ , showing that for the given numerical values of parameters  $m, n$  and  $s$ , the universe experiences an accelerated expansion.



**Figure 5.** Variation as a function of  $z$  of the Hubble parameter  $h(z)$  (left figure) and of the matter energy density  $r(z)$  (right figure) for a dust universe, for  $m = 0.002$ ,  $s = 0.20$ , and for different values of the parameters  $n$ :  $n = 1.653$  (solid curve),  $n = 1.663$  (dotted curve),  $n = 1.673$  (short dashed curve),  $n = 1.683$  (dashed curve), and  $n = 1.693$ , respectively.



**Figure 6.** Variation as a function of  $z$  of the deceleration parameter  $q(z)$  for a dust universe, for  $m = 0.002$ ,  $s = 0.20$ , and for different values of the parameters  $n$ :  $n = 1.653$  (solid curve),  $n = 1.663$  (dotted curve),  $n = 1.673$  (short dashed curve),  $n = 1.683$  (dashed curve), and  $n = 1.693$ , respectively.

#### 4 The special matter-aether coupling $\phi T$

In this Section we consider a special choice for the coupling between matter and the aether field. This coupling is common in massive gravity theories and galileons theories. In order to do this, we add a  $\alpha_6 \phi T$  term to the action Eq. (2.1), where  $\alpha_6$  is a coupling constant, and  $T$  is the trace of the matter energy-momentum tensor. One can easily see that this new coupling will add a new term  $\alpha_6 T$  to the left hand side of the aether equation of motion, Eq. (2.11), and a term of the form  $A_{\mu\nu}$  to the right hand side of the metric field equation Eq. (2.7), where

$$A_{\mu\nu} = \alpha_6 \left( \phi T_{\mu\nu} + \frac{1}{2} (T - 2\mathcal{L}_m) \phi g_{\mu\nu} + 2\phi g^{\alpha\beta} \frac{\partial^2 \mathcal{L}_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}} \right), \quad (4.1)$$

with  $\mathcal{L}_m$  as the matter Lagrangian (For more detailed calculations, see [27]).

We note that the energy-momentum tensor of the ordinary matter is not conserved due to the non-minimal interaction between scalar field and the energy-momentum tensor. In this case, the non-conservation equation can be obtained easily by taking the covariant

divergence of equation (2.7) and using equation (2.11), with the result

$$\nabla^\mu T_{\mu\nu} = \frac{\alpha_6}{1 + \alpha_6\phi} \left[ \phi^\mu (g_{\mu\nu} \mathcal{L}_m - T_{\mu\nu}) + \phi \nabla_\nu (\mathcal{L}_m - \frac{1}{2}T) - 2\nabla^\mu (\phi B_{\mu\nu}) \right], \quad (4.2)$$

where we have defined  $B_{\mu\nu} = g^{\alpha\beta} \frac{\partial^2 \mathcal{L}_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}}$ . The non-conservation of the energy-momentum tensor implies that the point particle does not follow the geodesic equation. In order to obtain the equation of motion for a point particle, we take assume that the energy-momentum tensor is described by a pressure-less perfect fluid, with

$$T_{\mu\nu} = \rho u_\mu u_\nu. \quad (4.3)$$

Substituting this into (4.2) and using the relation

$$h^{\nu\lambda} \nabla^\mu T_{\mu\nu} = u^\mu \nabla_\mu u^\lambda = \frac{d^2 x^\lambda}{ds^2} + \Gamma^\lambda_{\rho\sigma} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}, \quad (4.4)$$

we obtain the equation of motion as

$$\frac{d^2 x^\lambda}{ds^2} + \Gamma^\lambda_{\rho\sigma} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} = -\frac{h^{\nu\lambda}}{\rho} \frac{\alpha_6}{1 + \alpha_6\phi} \left[ \frac{1}{2} \phi \nabla_\nu \rho + \rho \phi_\nu \right]. \quad (4.5)$$

In the case  $\alpha_6 = 0$  we obtain the standard geodesic equation.

#### 4.1 The cosmological evolution in the presence of the $\phi T$ coupling

In order to obtain the effect of this new coupling term on the cosmological evolution of the universe, we note that one can choose the Lagrangian for a perfect fluid as  $\mathcal{L}_m = -\rho$ . Then for a flat Friedmann-Robertson-Walker geometry we obtain the metric and aether equations as

$$\frac{3}{2}(15\alpha_4 + 2\kappa^2)H^2 - 21\alpha_5 H^3 - 3\alpha_3 H - \rho + \lambda + \frac{\alpha_6}{2}(3p - \rho)t = 0, \quad (4.6)$$

$$(3\alpha_4 + 2\kappa^2 - 6\alpha_5 H)\dot{H} + \frac{3}{2}(3\alpha_4 + 2\kappa^2)H^2 - 6\alpha_5 H^3 + p + \frac{\alpha_6}{2}(5p + \rho)t = 0, \quad (4.7)$$

and

$$6(15\alpha_5 H^2 - 12\alpha_4 H + \alpha_3)\dot{H} + 90\alpha_5 H^4 - 108\alpha_4 H^3 + 18\alpha_3 H^2 - 2(3\lambda H + \dot{\lambda}) - \alpha_6(3p - \rho) = 0, \quad (4.8)$$

where we assume that  $c_1 = 0$  in Eq. (3.3).

As a first step in the analysis of the system of equations Eqs. (4.6) - (4.8) we rescale again the physical parameters according to

$$\begin{aligned} \alpha_3 &= \frac{2\kappa^2}{3} H_0 \eta, \quad \alpha_4 = \frac{2\kappa^2}{15} \sigma, \quad \alpha_5 = \frac{2\kappa^2}{21H_0} \theta, \quad H = H_0 h, \\ \rho &= 2\kappa^2 H_0^2 r, \quad p = 2\kappa^2 H_0^2 P, \quad \lambda = 2\kappa^2 H_0^2 \Lambda, \quad \alpha_6 = H_0 \Delta, \quad t = \frac{\tau}{H_0}, \end{aligned} \quad (4.9)$$

where  $H_0$  is the present day value of the Hubble parameter and  $\eta$ ,  $\sigma$  and  $\theta$  are dimensionless constants. Then the system of Eqs. (4.6) - (4.8) takes the following dimensionless form

$$\frac{3}{2}(1 + \sigma)h^2(\tau) - \theta h^3(\tau) - \eta h(\tau) - r(\tau) + \Lambda(\tau) + \frac{\Delta}{2}[3P(\tau) - r(\tau)]\tau = 0, \quad (4.10)$$

$$\left[1 + \frac{\sigma}{5} - \frac{2}{7}\theta h(\tau)\right] \frac{dh(\tau)}{d\tau} + \frac{3}{2}\left(1 + \frac{\sigma}{5}\right)h^2(\tau) - \frac{2}{7}\theta h^3(\tau) + P + \frac{\Delta}{2}[5P(\tau) + r(\tau)]\tau = 0, \quad (4.11)$$

$$6\left[\frac{5}{7}\theta h^2(\tau) - \frac{4}{5}\sigma h(\tau) + \frac{\eta}{3}\right] \frac{dh(\tau)}{d\tau} + \frac{30}{7}\theta h^4(\tau) - \frac{36}{5}\sigma h^3(\tau) + 6\eta h^2(\tau) - 2\left[3\Lambda(\tau)h(\tau) + \frac{d\Lambda(\tau)}{d\tau}\right] - \Delta[3P(\tau) - r(\tau)] = 0. \quad (4.12)$$

The above equations can not be solved analytically. Hence in the following we consider their numerical solutions for two particular choices of the equation of state of the cosmological matter.

#### 4.2 The radiation fluid cosmological model

In the case of a universe filled with a radiation fluid,  $P(\tau) = r(\tau)/3$ , the system of Eqs. (4.10) - (4.12) takes the form

$$r(\tau) = \frac{3}{2}(1 + \sigma)h^2(\tau) - \theta h^3(\tau) - \eta h(\tau) + \Lambda(\tau), \quad (4.13)$$

$$\begin{aligned} \frac{dh(\tau)}{d\tau} = \frac{1}{21(\sigma + 5) - 30\theta h(\tau)} & \left[ -35(4\Delta\tau + 1)\Lambda(\tau) + 35h(\tau)(4\Delta\eta\tau + \eta) \right. \\ & \left. + 5\theta(28\Delta\tau + 13)h^3(\tau) - 42h^2(\tau)[5\Delta(\sigma + 1)\tau + 2\sigma + 5] \right], \end{aligned} \quad (4.14)$$

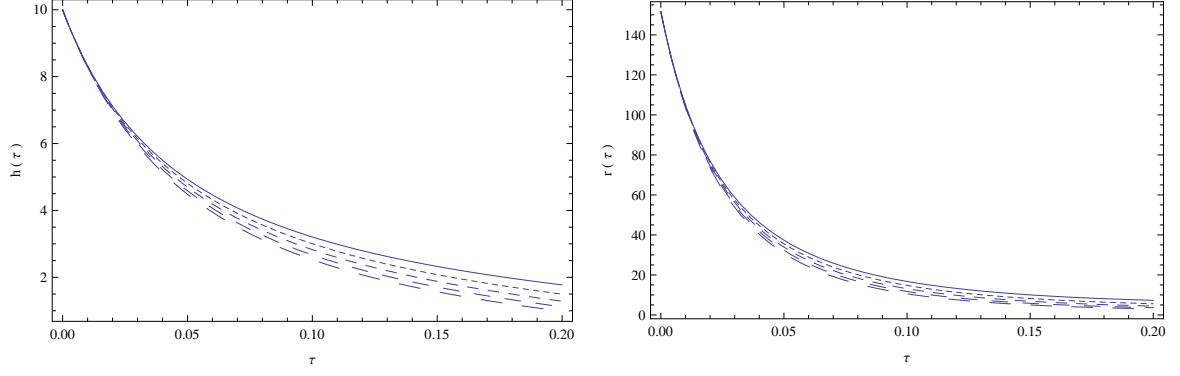
$$\begin{aligned} \frac{d\Lambda(\tau)}{d\tau} = \frac{1}{105[7(\sigma + 5) - 10\theta h(\tau)]} & \left\{ 35(4\Delta\tau + 1)\Lambda(\tau)[-35\eta - 75\theta h^2(\tau) + 84\sigma h(\tau)] \right. \\ & - 35h(\tau)(4\Delta\eta\tau + \eta) - 5\theta(28\Delta\tau + 13)h^3(\tau) + 42h^2(\tau)[5\Delta(\sigma + 1)\tau + 2\sigma + 5] \\ & \left. + 3\eta h^2(\tau) + \frac{15}{7}\theta h^4(\tau) - 3h(\tau)\Lambda(\tau) - \frac{18}{5}\sigma h^3(\tau) \right\}. \end{aligned} \quad (4.15)$$

The deceleration parameter can be obtained as

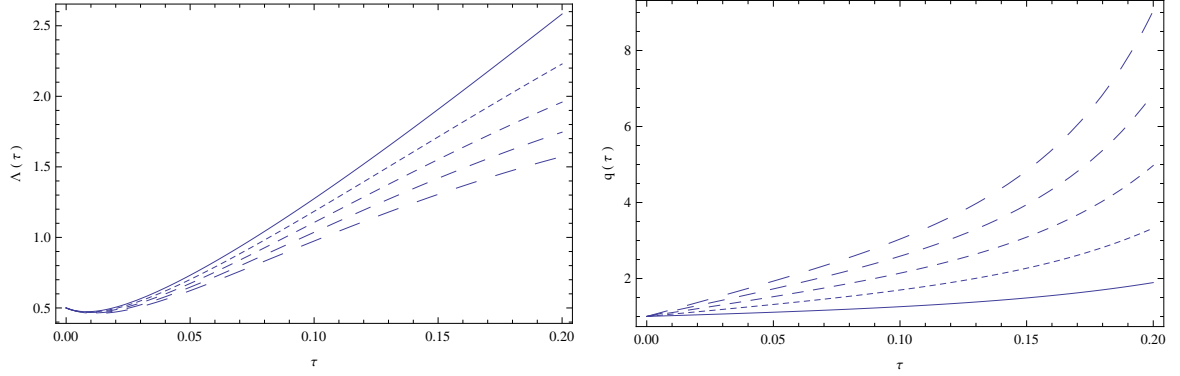
$$\begin{aligned} q(\tau) = \frac{1}{3h^2(\tau)(10\theta h(\tau) - 7(\sigma + 5))} & \left[ 7h(\tau)\left(5(4\Delta\eta\tau + \eta) + 5h^2(\tau)(4\Delta\theta\tau + \theta) \right. \right. \\ & \left. \left. - 3h(\tau)(10\Delta(\sigma + 1)\tau + 3\sigma + 5)\right) - 35(4\Delta\tau + 1)\Lambda(\tau) \right]. \end{aligned} \quad (4.16)$$

The time variations of the Hubble parameter, of the energy density, of the Lagrange multiplier, and of the deceleration parameter of the radiation fluid filled universe are presented, for different values of the parameter  $\Delta$ ,

in Figs. 7 and 8. In all cases  $\sigma = 0.001$ ,  $\theta = -0.001$ , and  $\eta = 0.001$ . The initial conditions used to numerically integrate Eqs. (4.14) and (4.15) are  $h(0) = 10$  and  $\Lambda(0) = 0.5$ .



**Figure 7.** Variation with respect to  $\tau$  of the Hubble parameter  $h(\tau)$  (left figure) and of the energy density  $r(\tau)$  (right figure) for a radiation fluid filled universe, for different values of the parameters  $\Delta$ :  $\Delta = 1$  (solid curve),  $\Delta = 3$ , (dotted curve),  $\Delta = 5$  (short dashed curve),  $\Delta = 7$  (dashed curve), and  $\Delta = 9$  (long dashed curve), respectively.



**Figure 8.** Variation with respect to  $\tau$  of the Lagrange multiplier  $\Lambda(\tau)$  (left figure) and of the deceleration parameter  $q(\tau)$  (right figure) for a radiation fluid filled universe, for different values of the parameters  $\Delta$ :  $\Delta = 1$  (solid curve),  $\Delta = 3$ , (dotted curve),  $\Delta = 5$  (short dashed curve),  $\Delta = 7$  (dashed curve), and  $\Delta = 9$  (long dashed curve), respectively.

In the plots we have adopted constant values for the parameters  $\sigma$ ,  $\theta$  and  $\eta$ , and we have varied the numerical value of  $\Delta$ , describing the strength of the coupling between  $\phi$  and  $T$ . As one can see from Fig. (7), the Hubble parameter is a monotonically decreasing function of time. The Lagrange multiplier  $\Lambda$ , presented in Fig. (8), has a complex cosmological evolution, consisting of two phases: a short, almost linearly decreasing with time which ends with reaching a minimum value at a finite time followed by a monotonic increase phase. The energy density  $r$ , shown in Fig. (7), is a monotonically decreasing function of time. As seen from Fig. 2.12, the dynamics of the deceleration parameter indicates that for all the chosen numerical values of the model parameters the radiation filled universe is in a decelerating phase. The time spent in the decelerating phase strongly depends on the numerical values of  $\Delta$ .

#### 4.3 The dust cosmological model

As a second example of a cosmological model in the framework of the SEA theory with aether-matter coupling we consider the case of a universe filled with dust, that is, matter with zero pressure. By taking  $P = 0$  the cosmological equations describing the evolutionary

dynamics of the universe become

$$r(\tau) = \frac{-\eta h(\tau) - \theta h^3(\tau) + (3/2)(\sigma + 1)h^2(\tau) + \Lambda(\tau)}{\Delta\tau/2 + 1}, \quad (4.17)$$

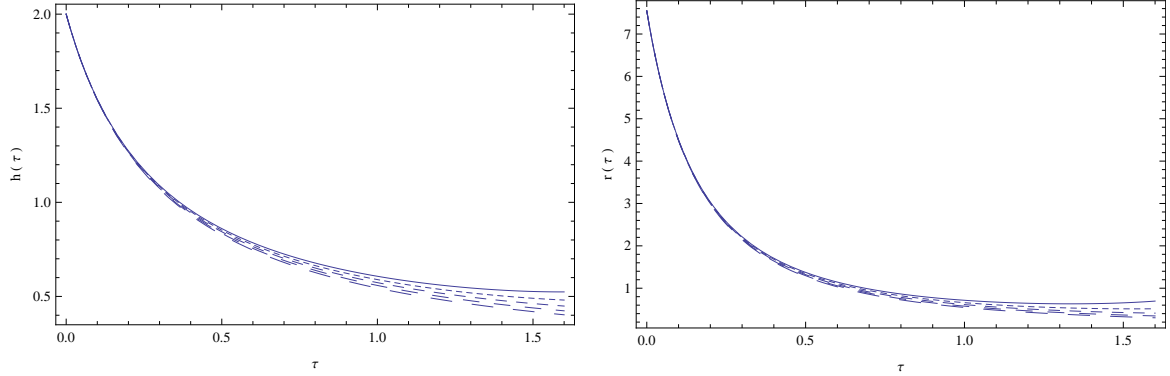
$$\frac{dh(\tau)}{d\tau} = \frac{-35\Delta\tau\Lambda(\tau) + 35\Delta\eta\tau h(\tau) + 5\theta(9\Delta\tau + 4)h^3(\tau) - 21h^2(\tau) [\Delta(3\sigma + 5)\tau + \sigma + 5]}{(\Delta\tau + 2) [7(\sigma + 5) - 10\theta h(\tau)]}, \quad (4.18)$$

$$\begin{aligned} \frac{d\Lambda(\tau)}{d\tau} = \frac{1}{2(\Delta\tau + 2) [7(\sigma + 5) - 10\theta h(\tau)]} & \left[ 14\Delta\Lambda(\tau)(-5\eta\tau + \sigma + 5) \right. \\ & + h^2(\tau)(\Delta(4\eta(5\theta - 63\sigma\tau) + 21(\sigma + 1)(\sigma + 5)) + 30\theta(4 - 3\Delta\tau)\Lambda(\tau) \\ & + 42\eta(\sigma + 5)) + 2h(\tau)(7\Delta\eta(5\eta\tau - \sigma - 5) - \Lambda(\tau)(\Delta(10\theta + 21(5 - 3\sigma)\tau) + 42(\sigma + 5))) \\ & + 4h^3(\tau)(9\Delta\tau(5\eta\theta + 7\sigma(\sigma + 1)) - \Delta\theta(11\sigma + 25) - 20\eta\theta) + 150\Delta\theta^2\tau h^5(\tau) \\ & \left. + 2\theta h^4(\tau)(10\Delta\theta - 6\Delta(32\sigma + 25)\tau + 9\sigma - 75) \right]. \end{aligned} \quad (4.19)$$

The deceleration parameter can be obtained as

$$q(\tau) = \frac{7h(\tau)(5\Delta\eta\tau - h(\tau)(2\Delta(4\sigma + 5)\tau - 5\Delta\theta\tau h(\tau) + \sigma + 5)) - 35\Delta\tau\Lambda(\tau)}{(\Delta\tau + 2)h^2(\tau)[10\theta h(\tau) - 7(\sigma + 5)]}. \quad (4.20)$$

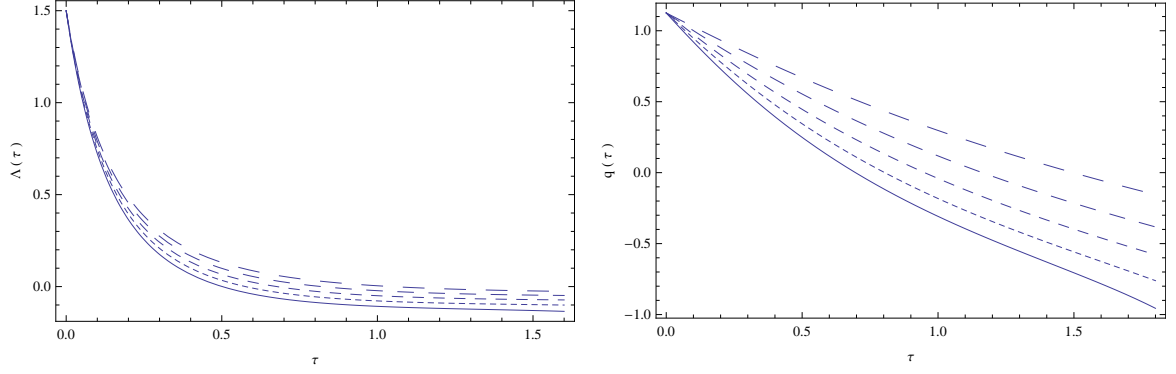
The time variation of the Hubble parameter  $h(\tau)$ , of the energy density  $r(\tau)$ , of the Lagrange multiplier  $\Lambda(\tau)$  and of the deceleration parameter  $q(\tau)$  are presented in Figs. 9 and 10.



**Figure 9.** Variation with respect to the dimensionless time  $\tau$  of the Hubble parameter  $h(\tau)$  (left figure) and of the matter energy density  $r(\tau)$  (right figure), for a dust universe, for different values of the parameters  $\Delta$ :  $\Delta = -0.75$  (solid curve),  $\Delta = -0.65$ , (dotted curve),  $\Delta = -0.55$  (short dashed curve),  $\Delta = -0.45$  (dashed curve), and  $\Delta = -0.35$  (long dashed curve), respectively.

Here, in a similar fashion to that of the radiation fluid filled universe model, we have also fixed the numerical values of the free parameters  $\sigma$ ,  $\theta$  and  $\eta$  as  $\sigma = 0.001$ ,  $\theta = -0.005$ ,  $\eta = 0.001$ , and have varied the coupling parameter  $\Delta < 0$ . The initial conditions used to integrate the field equations Eqs (4.18) and (4.19) are  $h(0) = 2$  and  $\Lambda(0) = 1.5$ , respectively. The Hubble parameter, depicted in Fig. 9, is a monotonically decreasing function of time, as well as the energy density, shown in Fig. 9. The Lagrange multiplier  $\Lambda$ , plotted in Fig. 10, shows





**Figure 10.** Variation with respect to the dimensionless time  $\tau$  of the Lagrange multiplier  $\Lambda(\tau)$  (left figure) and of the deceleration parameter  $q(\tau)$  (right figure) for a dust universe, for different values of the parameters  $\Delta$ :  $\Delta = -0.75$  (solid curve),  $\Delta = -0.65$ , (dotted curve),  $\Delta = -0.55$  (short dashed curve),  $\Delta = -0.45$  (dashed curve), and  $\Delta = -0.35$  (long dashed curve), respectively.

a similar behavior as for the radiation fluid cosmological model, decreasing monotonically with time. The deceleration parameter, depicted in Fig. 10, starts its evolution at a high redshift (high value of  $h$ ) with a positive value of the order of  $q \approx 1$ , indicating an initially decelerating cosmological evolution. Due to the presence of the aether field and of its coupling with matter the deceleration parameter monotonically decreases in time and the dust filled universe enters an accelerating stage, with the deceleration parameter reaching values of the order of  $q \approx -1$ .

## 5 Discussion and final remarks

In this paper we have studied a Lorentz violating theory of gravity by introducing a time-like vector field in the gravitational action. The time-like vector field is constructed from the covariant derivative of the scalar field  $\phi$ . In order to impose the time-like property of the vector  $\nabla_\mu \phi$ , we have added the constraint  $\nabla_\mu \phi \nabla^\mu \phi + 1 = 0$  to the action through a Lagrange multiplier. In order to add a kinetic term for the scalar field  $\phi$ , we note that the canonical kinetic term for the scalar field is already used in the constraint equation. With this constraint term, the scalar field becomes dynamical in the theory [26]. In order to enrich the dynamics of the scalar field one may add some higher order derivative scalar interaction terms to the action. This was done in [13] where the authors used the most general kinetic terms of a vector field to construct such interaction terms. However, the resulting theory suffers from ghost instabilities because of these higher order derivative interactions. This can be seen by noting that the theory [13] is equivalent to the projectable version of Horava-Lifshitz gravity which suffers from instabilities and strong coupling in low energies [7, 13, 16]. In this paper we have added some special higher order derivative interactions which produce second order field equation, and hence, no Ostrogradski instabilities. These interactions are known as Galileon terms [17].

The present model can also be considered as a generalization of the mimetic dark matter models [23]. In the mimetic dark matter model, one defines a physical metric out of a scalar field and some primary metric tensor in such a way that the physical metric becomes conformally invariant. Thus constructing a gravitational theory by the physical metric will automatically produce a conformally invariant gravitational theory. However, as was men-

tioned in [24], this theory is equivalent to a gravitational theory coupled to a scalar field and subject to the constraint that  $\nabla_\mu \phi$  is time-like. This is in fact equivalent to the scalar Einstein-aether theory [13]. In this sense, our present model is a generalization of the mimetic dark matter theory with some additional healthy derivative self-interaction terms. This will actually make the dark matter imperfect [25]. The problem of whether this model can satisfy dark matter experimental data will be considered elsewhere.

In the present paper, as a first step in the in-depth investigation of the model, we have studied the cosmological consequences of the Lorentz violating Galileon theory. If one assumes that the matter content of the universe consists of radiation or stiff matter, then the theory predicts a decelerating universe. In this case the universe decelerates more rapidly with stiff matter as compared to radiation. For a dust baryonic matter the model suggests an accelerating universe with an exponential de Sitter acceleration at late times.

We have also considered a special coupling between ordinary matter and the scalar field. This coupling will break the conservation law of the ordinary matter and hence one can expect some modification in the predictions of the Solar System tests, which may put a constraint on the value of the coupling constant  $\alpha_6$  in Eq. (4.1). This will also be done in a separate work. In this paper, we have considered only the cosmological implications of this extra term. In the radiation or stiff matter cases, the quantitative behavior of the universe does not change as compared to previous models without the interaction term, predicting a decelerating universe. In the case of a dust baryonic matter, however, one can obtain a universe starting from a decelerating phase and entering into an accelerating phase at later times. Other astrophysical and theoretical implications of this model such as black hole solutions and the stability of the cosmological solutions will be studied in a future publication.

## Acknowledgments

T. H. would like to thank the Department of Physics of the Sun-Yat Sen University in Guangzhou, P. R. China, for the kind hospitality offered during the preparation of this work.

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